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satisfied by the *four* values  $\theta=\lambda$ ,  $\theta=\mu$ ,  $\theta=\nu$ ,  $\theta=\rho$ , in virtue of the given equations; hence it must be an identity.

To find the value of  $x$ , multiply up by  $a+\theta$ , and then put  $a+\theta=0$ ; thus

$$x = \frac{(a+\lambda)(a+\mu)(a+\nu)(a+\rho)}{(a-b)(a-c)(a-d)}.$$

By symmetry, we have

$$y = \frac{(b+\lambda)(b+\mu)(b+\nu)(b+\rho)}{(b-c)(b-d)(b-a)},$$

$$z = \frac{(c+\lambda)(c+\mu)(c+\nu)(c+\rho)}{(c-d)(c-a)(c-b)},$$

$$\text{and } u = \frac{(d+\lambda)(d+\mu)(d+\nu)(d+\rho)}{(d-a)(d-b)(d-c)}.$$

Similarly solved by G. B. M. ZERR.

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### GEOMETRY.

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185. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Given the tangential equations to two conics  $S, S'$ , find the tangential co-ordinates of the join of the poles of two given parallel lines with respect to  $S$ . Deduce the tangential equation of the center of  $S$ , and find that of the intersection of  $S$  and  $S'$ .

Solution by G. B. M. ZERR, A.M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let  $bc-f^2=A$ ,  $ca-g^2=B$ ,  $ab-h^2=C$ ,  $gh-af=F$ ,  $hf-bg=G$ ,  $fg-ch=H$ .

Then  $S=A\lambda^2+B\mu^2+C\nu^2+2F\mu\nu+2G\nu\lambda+2H\lambda\mu$ .

Similarly,  $S'=A'\lambda^2+B'\mu^2+C'\nu^2+2F'\mu\nu+2G'\nu\lambda+2H'\lambda\mu$ .

Let  $\lambda a+\mu\beta+\nu\gamma$  and  $\lambda a+\mu\beta+\nu\gamma+m$  be the two given parallel lines;  $p, q, t$  and  $p', q', t'$  their poles with respect to  $S$ . Then for the first line

$$ap+hq+gt=\lambda, \quad hp+bq+ft=\mu, \quad gp+fq+ct=\nu.$$

Solving these equations for  $p, q, t$ ,

$$p=(A\lambda+H\mu+G\nu)/\Delta, \quad q=(H\lambda+B\mu+F\nu)/\Delta,$$

$$t=(G\lambda+F\mu+C\nu)/\Delta, \quad \text{where } \Delta=abc+2fgh-af^2-bg^2-ch^2.$$

For the second line,

$$ap'+hq'+gt'+m=\lambda, \quad hp'+bq'+ft'+m=\mu, \quad gp'+fq'+ct'+m=\nu.$$

$$\therefore p' = [A\lambda + H\mu + G\nu - m(A + H + G)] / \Delta.$$

$$q' = [H\lambda + B\mu + F\nu - m(H + B + F)] / \Delta.$$

$$t' = [G\lambda + F\mu + C\nu - m(G + F + C)] / \Delta.$$

$(p, q, t)$ ,  $(p', q', t')$  are the tangential co-ordinates of the join of the poles.

Let  $A', B', C'$  be the angles of the triangle of reference. The center is the pole of the line at infinity  $\alpha \sin A' + \beta \sin B' + \gamma \sin C' = 0$ . The tangential co-ordinates of the center are obtained by substituting  $\sin A', \sin B', \sin C'$  for  $\lambda, \mu, \nu$  in  $p, q, t$  and are

$$S_1 = (A \sin A' + H \sin B' + G \sin C') / \Delta,$$

$$S_2 = (H \sin A' + B \sin B' + F \sin C') / \Delta,$$

$$S_3 = (G \sin A' + F \sin B' + C \sin C') / \Delta.$$

$\therefore$  The tangential equation of the center is  $\lambda S_1 + \mu S_2 + \nu S_3 = 0$ .

Write  $a + ka'$  for  $a$ ,  $b + kb'$  for  $b$ ,  $c + kc'$  for  $c$ ,  $f + kf'$  for  $f$ ,  $g + kg'$  for  $g$ ,  $h + kh'$  for  $h$  in  $A\lambda^2 + B\mu^2 + C\nu^2 + 2F\mu\nu + 2G\nu\lambda + 2H\lambda\mu = 0$ .

Then the tangential equation of the four points of intersection of  $S$  and  $S'$  is  $S + k\Phi + k^2S' = 0$  where  $k$  is undetermined, and

$$\begin{aligned} \Phi = & (bc' + b'c - 2ff')\lambda^2 + (ca' + c'a - 2gg')\mu^2 + (ab' + a'b - 2hh')\nu^2 \\ & + 2(gh' + g'h - af' - a'f)\mu\nu + 2(hf' + h'f - bg' - b'g)\nu\lambda \\ & + 2(fg' + f'g - ch' - c'h)\lambda\mu. \end{aligned}$$

The condition for equal roots for  $k$  is  $\Phi^2 = 4SS'$ , which is the equation of the four points of intersection.

186. Proposed by J. R. HITT; Professor of Mathematics, Coronal Institute, San Marcos, Texas.

If two sides of a triangle and its in-circle be given in position, the envelope of its circumscribed circle is a circle (*Mannheim*). [From Casey's *Sequel to Euclid*.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Let vertex  $A$  be origin, sides  $b, c$  the axes. Then  $x^2 + 2xy \cos A + y^2 - bx - cy = 0$  is the equation to the circumscribed circle. Let this equation be written

$$D - bx - cy = 0 \dots (1).$$

Since the sides  $b, c$  and the inscribed circle are fixed in position, the tangents from  $A$  to the in-circle are constant.

$$\therefore b + c - a = \text{a constant} = m \dots (2).$$

$a = \sqrt{(b^2 + c^2 - 2bc \cos A)}$ . This in (2) gives after reduction,

$$m^2 + 2bc(1 + \cos A) - 2m(b + c) = 0 \dots (3).$$